Viscous Spacetime Fluid and Higher Curvature Gravity

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The Einstein field equation as an equation of state of a thermodynamical system of spacetime is reconsidered in the present Letter. We argue that a consistent interpretation leads us to identify scalar curvature and cosmological constant terms representing the bulk viscosity of the spacetime fluid. Since Einstein equation itself corresponds to a near-equilibrium state in this interpretation invoking $f(R)$ gravity for nonequilibrium thermodynamics is not required. A logically consistent generalization to include the effect of so called 'tidal forces' due to the Riemann curvature is presented. A new equation of state for higher curvature gravity is derived and its physical interpretation is discussed.

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Higher curvature Lagrangians have been discussed as logically possible constructions for gravitation since long [1] , however current interest in such theories is inspired by various shades of quantum gravity/effective field theory [2]. Amongst them $f(R)$ gravity, where f is a nonlinear function of the scalar curvature R, is considered as an attractive model to explain the observed cosmic acceleration phase [3]. The conflict of the metric formulation of $f(R)$ gravity with the solar system observations has been claimed to get resolved in the Palatini formalism [4], however it has been argued that this theory results in the appreciable deviations from the microphysics (e.g. electron-electron scattering experiments), and violation of the equivalence principle [5]. It may be asked: Is there an alternative approach to higher curvature gravity?

Departing from the variational principle approach, recently a thermodynamical derivation of the $f(R)$ gravity has been proposed [6] generalizing the previous derivation of the Einstein field equation [7]. Thermodynamics of spacetime in [7] is motivated by the black hole thermodynamics: it is assumed that similar to the black hole entropy formula a universal entropy density α per unit horizon area for all local Rindler horizons could be defined, and the Clausius relation $TdS = \delta Q$ of equilibrium thermodynamics holds for all local horizons. Here the heat flow across the horizon δQ is defined to be the boost energy of the matter and T is the Unruh temperature. Jacobson then argues that since entropy is interpreted as horizon area the Clausius relation would be satisfied provided the spacetime curvature in the presence of matter is such that the Einstein field equation holds. In [6] this approach is applied to the assumed entropy density proportional to $f(R)$, and it is found that local energy conservation entails entropy production term, and hence nonequilibrium thermodynamics of spacetime. Further the higher curvature equation of state obtained is shown to be identical with the field equation derived from the $f(R)$ Lagrangian.

Obviously rederivation of the known field equations by itself is not of much significance unless new insights are gained. Unfortunately the paradigm of field theory has overshadowed this promising thermodynamic approach in [6, 7]. Moreover we have shown [8] that covariant divergence law for the matter stress tensor is satisfied in a conformally related spacetime, $\tilde{g}_{\mu\nu} = f(R)g_{\mu\nu}$ with no need to postulate bulk viscosity production term as is done by Eling et al in [6], and therefore the issue of nonequilibrium thermodynamics becomes trivial. Note that this result is independent of the unimodular perspective suggested in [8].

In this Letter spacetime as a thermodynamical system in the spirit of Jacobson's approach is considered with the two-fold aim: 1) to gain new insights into the nature of spacetime assumed to be some kind of fluid at large scale, and 2) to motivate nonequilibrium thermodynamics in the case considered in [7] and generalize it to include the effect of the so called 'tidal force' caused by the Riemann curvature of spacetime [9] and derive a new higher curvature equation of state.

First we argue that a consistent equation of state interpretation of the Einstein field equation demands a careful reanalysis of the local equilibrium condition: the Φ term in Eq.(6) below while easily interpreted in the field theoretic setting, seems to require a different meaning since local energy conservation is assumed to be a form of first law of thermodynamics [6]. This term in the Einstein field equation in the absence of matter stress tensor is suggested to represent a dissipative contribution to the stress tensor of vacuum. Analogous to the bulk viscosity for the dissipative fluid [10], Φ term, therefore signifies viscous spacetime, and the Einstein equation already corresponds to a quasi-equilibrium system.

Next we recall Bondi's argument [11] noted in [8] that the essence of observable gravitation is that relative acceleration varies at each spacetime point. It would mean that even the assumption of local thermodynamic equilibrium for local Rindler horizons would, in principle, fail. However the gravitation is so weak that the correction due to Riemann curvature is proposed to be incorporated retaining key elements of the Jacobson's approach in the following. The notion of local Rindler horizon is used to define heat flow and entropy change in [7]. The equivalence principle enables locally flat spacetime at each spacetime point, and for an infinitesimal 2-surface ele-

ment through the point one can assume vanishing shear and expansion to the past (or inside) of the plane. The past horizon, the local Rindler horizon is assumed instantaneously stationary. This idea helps in introducing an approximate local boost Killing vector field which can be related with the horizon tangent vector and affine parameter along the null geodesic. Now instead of employing the Jacobi deviation equation for relative acceleration, we assume that the imprint of the tidal force is carried by the congruence of the null geodesics via a correction term dependent on the Riemann curvature. Both nonuniform temperature and viscosity lead to dissipation; since we do not know how to define these quantities here instead of the entropy change we calculate change in the thermodynamic potential $F = ST$. Heat flow and change in the horizon area are calculated for local Rindler horizons to lowest order in the affine parameter as is done in [7]. A correction term proportional to the Riemann curvature is introduced in the change δF. Equating δQ and δF, and imposing covariant divergence law for matter stress tensor finally give the desired equation of state. The Letter concludes with a brief discussion on the physical interpretation and prospects of this equation.

Briefly the main ideas of [7] are as follows. The black hole formula, namely the proportionality between entropy and the horizon area, is assumed to hold for all local Rindler horizons at each spacetime point of the manifold M. Causal horizon at a point p is specified by a space-like 2-surface B, and the boundary of the past of B comprises of the congruences of null geodesics. Assuming vanishing shear and expansion at p the past horizon of B is called local Rindler horizon. The energy flux across the horizon is used for heat energy, and calculated in terms of the boost energy of matter: define an approximate boost Killing vector field χ^μ future pointing on the causal horizon, and related with the horizon tangent vector k^μ and affine parameter λ by $\chi^\mu = -a\lambda k^\mu$. The heat flux to the past of B is given by

$$\delta Q = \int T_{\mu\nu}\chi^\mu d\Sigma^\nu \tag{1}$$

The integral is taken over a small region of pencil of generators of the inside past horizon terminating at p. If area element is dA then $d\Sigma^\nu = k^\nu d\lambda dA$, and Eq.(1) becomes

$$\delta Q = -a\int T_{\mu\nu}k^\mu k^\nu \lambda d\lambda dA \tag{2}$$

Change in the horizon area is given in terms of the expansion of the congruence of null geodesics generating the horizon $\delta A = \int \theta d\lambda dA$. The expansion of the null geodesics generating the horizon is given by the Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu \tag{3}$$

Assuming vanishing shear and neglecting θ^2 term we get the solution

$$\theta = -\lambda R_{\mu\nu}k^\mu k^\nu \tag{4}$$

Assuming universal entropy density α per unit horizon area it is straightforward to calculate the entropy change

$$\delta S = -\alpha\int R_{\mu\nu}k^\mu k^\nu \lambda d\lambda dA \tag{5}$$

The condition that the Clausius relation is satisfied for all null vectors k^μ , and making use of the Unruh temperature $\hbar a/2\pi$ gives

$$R_{\mu\nu} + \Phi g_{\mu\nu} = (2\pi/\hbar\alpha)T_{\mu\nu} \tag{6}$$

The unknown function Φ is determined using the covariant divergence law for the stress tensor and contracted Bianchi identity; Einstein equation with a cosmological constant (CC) Λ is obtained. Here Newton's gravitational constant is identified as $G = 1/4\hbar\alpha$, and the function Φ is obtained to be

$$\Phi = -\frac{R}{2} + \Lambda \tag{7}$$

In the light of thermodynamic derivation a logically consistent physical interpretation of the equation of state is proposed here. In the Clausius relation integrands of (2) and (5) have been equated for all null vectors to obtain Eq.(6). Since $T_{\mu\nu}$ is the matter stress tensor, from the integral form itself it is logical to infer that $R_{\mu\nu}$ is proportional to the stress of assumed spacetime fluid. What does Φ term represent? In the derivation Φ term is needed to satisfy the first law of thermodynamics (in the form of local energy conservation), and hence it should correspond to an additional stress tensor for a dissipative process in Eq.(6). Noting its formal similarity with the bulk viscosity stress tensor for an isotropic fluid proportional to the tensor δ_{ij}, see Eq.(8.4.42) in [10] it seems reasonable to identify Φ term in Eq.(6) as bulk viscosity tensor of the spacetime. Deeper insight is gained analysing the CC term. Setting matter stress tensor zero and assuming a vanishing CC the Ricci flat spacetime becomes indistinguishable from $G_{\mu\nu} = R_{\mu\nu} - \frac{R}{2}g_{\mu\nu} = 0$ as $R = 0$. Thus vacuum Einstein equation with vanishing CC could be interpreted as an equation of state of a perfect fluid. The presence of matter is suggested to cause a dissipative process leading to a nonvanishing scalar curvature, i.e. a bulk viscosity to the spacetime; geometrically scalar curvature arises via the contracted Bianchi identity so that the Einstein tensor satisfies the covariant divergence law. Here R has been given a thermodynamic significance.

What is the physical significance of Λ ? In the light of its appearance in combination with R in (7) this would correspond to the bulk viscosity treating the whole Φ term as the bulk viscosity stress tensor of the spacetime. Thus empty spacetime with nonvanishing CC would be like a viscous spacetime fluid. In Jacobson's approach the thermal behavior of quantum vacuum in flat spacetime is an important ingredient. The time translation symmetry in Rindler wedge corresponds to the boost symmetry of the Minkowskian spacetime; quantum vacuum in

Minkowskian spacetime as observed in the uniformly accelerating frame acquires thermal properties of a thermal bath of real particles with the Unruh temperature [12]. Assuming that quantum vacuum fluctuations would be significant only at Planck length scales a universal entropy per unit horizon area and Unruh temperature have been assumed in [7]. This is a reasonable assumption, however in the light of thermodynamic derivation it may be asked: Does there exist averaged out observable effect of vacuum fluctuations? There is an interesting phenomenon in quantum optics [13] : atomic motion in light radiation in the so called optical molasses bears resemblance with that of a particle in a viscous fluid. Analogous to this, envisaging gravitational molasses having origin in quantum fluctuations in the spacetime bulk viscosity associated with CC could be attributed to this effect. Another possibility is, following suggestions in the literature, to treat CC as the energy density of quantum vacuum fluctuations, $\Lambda = 8\pi G V_0/c^4$ where V_0 is the vacuum expectation value. In this case, nonzero CC would cause dissipative process similar to matter energy and constant scalar curvature would determine the bulk viscosity of spacetime. We emphasize that both CC and quantum vacuum possess rather speculative character in general relativity, and the microscopic nature of the spacetime is unknown to us, hence the preceding discussion though quite plausible, also remains speculative.

The question of bulk viscosity for the Einstein equation is also discussed in [6]; the expected bulk viscosity of $3\hbar\alpha/4\pi$ is obtained from the $f(R)$ equation in the Einstein frame. Authors argue it to be incorrect, and note the ambiguity of sign. However negative bulk viscosity could be related with the acausal teleological boundary conditions for the black hole [14]. The stretched-horizon formalism developed in [14] gives useful insights on black hole physics; in particular, energy and momentum conservation laws for the membrane resemble with those of viscous fluid. The present interpretation that the Einstein equation represents a viscous spacetime fluid could be viewed as a generalization, albeit a radical one, of the membrane paradigm of [14].

Thermodynamic derivation of the Einstein equation based on arbitrary spacelike 2-surfaces has been recently established [15]. It is, therefore, reasonable to assert that the preceding physical interpretation is of general validity. Clearly viscosity term implies that the system is not in thermodynamic equilibrium; moreover higher curvature, e.g. $f(R)$ gravity, is not needed to motivate nonequilibrium thermodynamics. The present interpretation however does offer an effective procedure to include the effect of Riemann curvature as a correction to viscous stress tensor. Returning to the fluid analogy, in general the dissipative coupling coefficient η_{ijkl} is a fourth rank viscosity tensor [10], and has symmetry of indices similar to curvature tensor $R_{\mu\nu\alpha\beta}$. Assuming that the correction due to the Riemann curvature is proportional to $R_{\mu\sigma\nu\rho}$ the product with the Ricci tensor keeping in mind the symmetry of the indices would be $R^{\sigma\rho}R_{\mu\sigma\nu\rho}$. Thus we construct the simplest form of δF

$$\delta F = -\frac{\alpha\hbar a}{2\pi}\int [R_{\mu\nu} + \beta(R^{\sigma\rho}R_{\mu\sigma\nu\rho} - \frac{R}{2}R_{\mu\nu})]k^\mu k^\nu \lambda d\lambda dA \tag{8}$$

The last term in the square brackett in the integrand corresponds to the viscosity as identified above in Eq.(7) for the scalar curvature. This also makes the application of covariant divergence law unambiguous. Using Clausius relation and equating the integrands of (8) and (2) for all null vectors we get

$$R_{\mu\nu} + \beta(R^{\sigma\rho}R_{\mu\sigma\nu\rho} - \frac{R}{2}R_{\mu\nu}) + \Psi g_{\mu\nu} = \frac{2\pi}{\hbar\alpha}T_{\mu\nu} \tag{9}$$

The unknown function Ψ is determined taking the covariant divergence of (9) and using the Bianchi identity and vanishing divergence of matter stress tensor. Use is made of following relations

$$(R_{\mu\sigma\nu\rho}R^{\sigma\rho})^{;\mu} = (\frac{1}{4}R_{\sigma\rho}R^{\sigma\rho} - \frac{1}{2}R_{;\alpha}^{\ ;\alpha})_{,\nu} + \frac{1}{2}(R_{;\mu\nu})^{;\mu} \tag{10}$$

$$(RR_{\mu\nu})^{;\mu} = (R_{;\mu\nu})^{;\mu} + (\frac{R^2}{4} - R_{;\alpha}^{\ ;\alpha})_{,\nu} \tag{11}$$

The expression for Ψ is finally obtained to be

$$\Psi = -\frac{R}{2} + \frac{\beta R^2}{8} - \frac{\beta R^{\sigma\rho}R_{\sigma\rho}}{4} \tag{12}$$

Substituting Ψ in (9) we get the desired higher curvature equation of state

$$G_{\mu\nu} - \frac{\beta R}{2}(R_{\mu\nu} - \frac{R}{4}g_{\mu\nu}) + \beta R^{\sigma\rho}(R_{\mu\sigma\nu\rho} - \frac{R_{\sigma\rho}}{4}g_{\mu\nu}) = \frac{2\pi}{\hbar\alpha}T_{\mu\nu} \tag{13}$$

Perusal of the higher curvature field equations in the literature [2, 3, 4] shows that Eq.(13) is a new result. This equation has some remarkable properties: 1) The arbitrary constant β could be adjusted to keep intact the established physical tests of the Einstein equation since the higher curvature effect appears as a correction to $G_{\mu\nu}$. 2) The trace of (13) leads to the same result as that of pure general relativity. Note also that the first two terms are same as the ones in R^2 gravity in the Palatini formalism which would make it to be in agreement with some of the results of that theory. However the third term is a new addition. And 3) In contrast to the higher curvature field equations obtained from the action principle which are fourth order derivative equations, Eq.(13) is a second order derivative equation similar to that one obtains in the Palatini version where the action is varied taking metric and affine connection as independent variables. These characteristics strongly suggest viability of Eq.(13) as higher curvature gravity theory.

It is well known that the question of gravitational energy in general relativity is quite complex [16], and even

more so in higher curvature gravities [2, 17]. An interesting consequence of (13) is that in the empty spacetime it reduces to

$$R_{\mu\nu} + \beta R^{\sigma\rho}(R_{\mu\sigma\nu\rho} - \frac{R_{\sigma\rho}}{4} g_{\mu\nu}) = 0 \tag{14}$$

What is the physical significance of the second term in Eq.(14)? In analogy with the traceless stress tensor of electromagnetic field it is plausible to identify this term to represent stress tensor of gravity radiation loss. Bondi presented a lucid discussion [18] on the inductive and wave transfer of gravitational energy in general relativity, and considered axially-symmetric vacuum solution of Weyl and Levi-Civita for this purpose. Since the vacuum defined by Eq.(14) has different nature, it would be interesting to delineate inductive and wave energy transfers in this case.

The problem of entropy production for $f(R)$ gravity has been extensively discussed in [19]. What would be the entropy production in the present case? There is another interesting question: Could one derive Eq.(13) in the action formalism? We propose to investigate Palatini formalism for the action used by Deser and Tekin [2] elsewhere since the main theme of the present work is thermodynamic approach to the spacetime.

In conclusion, we have argued that Einstein equation in thermodynamic approach represents a viscous spacetime fluid, and derived a new equation of state for higher curvature corrections.

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